OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Unit **Y534**: Discrete Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

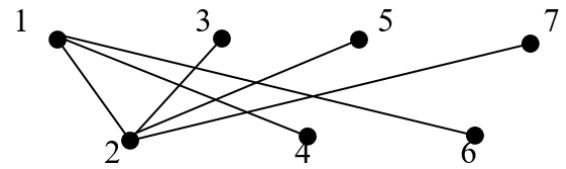
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		Crate 1: 12 small jars Crate 2: 5 small and 1medium jar Crates 3 and 4: 3 medium jars each Crates 5, 6 and 7: 1 large jar each	 M1 A1 [2]	 1.1 1.1	 Crates 1 and 2 correct All correct, in this order	May use S, M, L provided intention is obvious If using, e.g. 4S for M, must have defined this for A1
1	(ii)		E.g. Crates 1, 2and 3: 1 large jar and 3 small jars each Crates 4 and 5: 3 medium jars each Crate 6: 1 medium and 8 small jars There is no spare capacity/six full crates /all full	 M1 A1 B1 [3]	 3.1b 1.1 2.4	 Attempt a packing using 6 crates o.e. Pack 17 small, 7 medium and 3 large in 6 boxes, in any order $(17 + 7\times4 + 3\times9)\div12 = 72\div12 = 6$	For reference: Crate size = 12 Medium = ×4, large = ×9
1	(iii)		e.g. Put each large jar in a box, then put two medium jars in boxes until no more pairs are possible. Put any remaining medium jar in another box and fill in the spaces with small jars.	E1 [1]	3.1b	Description of a general minimising strategy that has at most 2 medium jars in any box	
2	(i)		There must be at least one day on which Mo eats at least two doughnuts	B1 [1]	1.1	Or equivalent	
2	(ii)		The maximum on any one day is 3 any day next to the 3 must be a 1, 3+1 = 4 Or 2 on two days and 1 on two days If the 2's are on adjacent days this gives 4 If not there is a 2 on (at least one) end so the other three days total 4	E1 M1 A1 [4]	2.1 1.1 3.1b	1 1 <u>1 3</u>, 1 <u>1 3</u> 1, <u>1 3</u> 1 1, <u>3 1</u> 1 1 Start to consider cases for 2 2 1 1 May be implied from working May list and identify where total = 4 Dealing with both 2 2 1 1 situations and describing or showing 4's	 <u>2 2</u> 1 1, 1 <u>2 2</u> 1 or 1 1 <u>2 2</u> <u>2 1 1</u> 2, <u>1 2 1</u> 2 or 2 <u>1 2 1</u>
2	(iii)		4! ÷ 2! =12 arrangements of 0, 0, 1, 2 4 arrangements of 0, 1, 1, 1 4 arrangements of 0, 0, 0, 3	B1 B1 B1 [3]	2.4 1.1 1.1	Or $^4C_2 \times 2 = 12$ or write out these 12 Or write out these 4 Or write out these 4	

Question			Answer	Marks	AO	Guidance	
3	(i)		<table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>X</td><td>(1, 4)</td><td>(5, 3)</td><td>(2, 6)</td></tr><tr><td>Y</td><td>(5, 2)</td><td>(1, 3)</td><td>(0, 1)</td></tr><tr><td>Z</td><td>(4, 3)</td><td>(3, 1)</td><td>(2, 1)</td></tr><tr><td>Worst for cols</td><td>2</td><td>1</td><td>1</td></tr></table>	**M1**	**3.1a**	Using worst pay-off in column(s) (2, 1, 1)	Worst result in column P (for player on columns) is 2
			2 > 1 so P is better than Q and R	**A1** **[2]**	**3.2a**	Maximin (in column P)	Explaining why P (as given)
3	(ii)		e.g. If player on columns plays safe (P) then player on rows does best by playing Y (1, 5, 4). But then player on columns does better by changing from P to Q.	**M1** **A1** **[2]**	**2.1** **2.2a**	Play-safe for player on rows is Z (1, 0, 2) (Y, P) has a better pay-off for the player on rows than (Z, P)	 Or cell (P, Z) ≠ (2, 2) i.e. the play-safe values
3	(iii)		A = {y : $y \leq 4$}	**B1** **[1]**	**1.1**	Or equivalent Allow, e.g. A ≤ **4**	Allow $y < 4$ or $y \leq 3$ But not just a single specific value
3	(iv)		In row Z, column P has the best pay-off for the player on columns	**B1** **[1]**	**2.4**	Row Z: 3 > 1 (for player on cols)	For the player on cols P > Q in row X (or Z) and P > R in row Z

Question			Answer	Marks	AO	Guidance	
4	(i)		12	B1 [1]	1.2	cao	
4	(ii)		Must start at one of 1, 3, 5, 7 4 choices × 3 choices × 3 choices × 2 choices × 2 choices = 144 But each path can be travelled in either direction So 144 ÷ 2 = 72	B1 M1 A1 [3]	1.1 2.5 2.4	May be implied from working Using multiplicative principle 4! × 3! (=144) 72, from correct working seen	Allow 3×3×2×2×1×1 (=36) ⇒ B1 M1 144 with no working or incorrect working ⇒ B0 M1
4	(iii)	(a)	e.g. Using Kruskal's algorithm 1×2 = 2 1×4 = 4 1×6 = 6 and 2×3 = 6 2×5 = 10 ~~3×4 = 12~~ forms a cycle 2×7 =14 …  	B1 B1 B1 [3]	1.1 1.1 1.1	Arc weights correct [at least six correct arc weights shown in a list or on a diagram] Evidence of using Kruskal or Prim [K: need not list all arcs] [P: may start at any vertex] These six arcs chosen (list or tree)	
4	(iii)	(b)	42	B1 [1]	1.1	cao	

Question			Answer	Marks	AO	Guidance	
5	(i)		One luxury pack would not give enough cards with flowers. One standard or economy would not give enough handmade cards.	B1 B1 [2]	2.2a 2.2a	Explaining why one luxury pack is not enough Explaining why one standard pack or one economy pack is not enough No pack has 8 handmade cards and 8 cards with flowers	One luxury pack only has 20 cards (Alice needs 25 cards) One standard pack only has 5 handmade cards and an economy pack has none (Alice needs 8)
5	(ii)	(a)	1 luxury and 1 standard 1 luxury and 1 economy 2 standard	M1 A1 [2]	1.1 1.1	Any one of these All three and no others (or any extras have been rejected)	In any form, e.g. LS or SL
5	(ii)	(b)	2 standard	B1 [1]	1.1		
5	(iii)		Two luxury packs satisfies all the requirements but costs £13. However Ben will buy back 15 cards. Ben can buy back up to 12 handmade cards (12 × 12p) + (3×5p) = £1.59 Giving net cost £11.41 Or a valid calculation, with Ben buying 15 cards, with explanation of how many of each type he bought.	M1 A1 [2]	1.1 1.1	Description or calculation showing which surplus cards Ben buys back. e.g. calc leading to one of 11.48, 11.55, 11.62, 11.69. 11.76 (in any form)	e.g. £13 and a multiple of 12p or a multiple of 5p or a combination of both (shown) e.g. calc leading to one of 11.56, 11.68, 11.80, 11.92 (in any form)

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th colspan="2">Guidance</th></tr>
<tr><td>5</td><td>(iv)</td><td></td><td>Handmade: $10x + 5y$
Flowers: $5x+10y+10(2 – x – y) = 20 – 5x$
Animals: $5x+5y+10(2 – x – y) = 20 – 5x – 5y$
Other: $10y+20(2-x-y) = 40-20x-10y$</td><td>B1
M1
A1

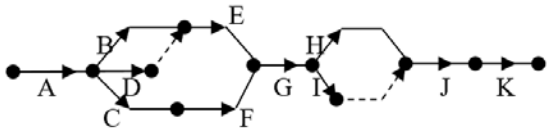
A1
[4]</td><td>1.1
3.1b
1.1

1.1</td><td>Correct expression for handmade
Using $2 – x – y$ appropriately
Correct expression for one of flowers, animals, other
All of these three correct, simplified</td><td>Or flowers = $5x + 10(2 – x)$</td></tr>
<tr><td>5</td><td>(v)</td><td></td><td>$650x + 500y + 400(2 – x – y)$
$–12(10x+5y–8) – 5(10x+25y+40(2 – x – y) – 17)$
$= 581 + 280x + 115y$</td><td>M1
M1
A1
[3]</td><td>1.1
3.1b
1.1</td><td>$250x + 100y + 800$
$30x + 15y – 219$
cao</td><td>Accept $650x + 500y$
Accept $-12(…) – 5(…)$</td></tr>
<tr><td>5</td><td>(vi)</td><td></td><td>Min when x is as small as possible and then y is as small as possible whilst still satisfying Alice’s requirements
$x = 0$ and $y = 2$

£8.11</td><td>M1

A1
[2]</td><td>1.1

1.1</td><td><table><tr><th>x</th><th>y</th><th>$581 + 280x + 115y$</th></tr><tr><td>0</td><td>2</td><td>811</td></tr><tr><td>1</td><td>0</td><td>861</td></tr><tr><td>1</td><td>1</td><td>976</td></tr><tr><td>2</td><td>0</td><td>1141</td></tr></table>cao</td><td>Using their objective
Or a fresh start</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>6</td><td>(i)</td><td></td><td> 

Forward pass
Minimum project completion time = 10 days

Backward pass
Critical activities: A, B, E, G, H, J, K

<table><tr><td>Activity</td><td>C</td><td>D</td><td>F</td><td>I</td></tr><tr><td>Float (days)</td><td>1</td><td>0.5</td><td>1</td><td>0.5</td></tr></table></td><td>B1

M1
A1

M1
A1

M1
A1
[7]</td><td>3.3

1.1
3.4

1.1
3.4

1.1
3.4</td><td>Activity network attempted

At least one merge correct
10

At least one burst correct
A, B. E. G, H, J, K

Any one float correct
All correct</td><td>For reference: durations
<table><tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr><tr><td>2</td><td>1</td><td>1</td><td>0.5</td><td>2</td><td>1</td></tr></table>
<table><tr><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td></tr><tr><td>1</td><td>1</td><td>0.5</td><td>2</td><td>1</td></tr></table>
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr><tr><td>ES</td><td>0</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>LF</td><td>2</td><td>3</td><td>4</td><td>3</td><td>5</td></tr></table>
<table><tr><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td></tr><tr><td>3</td><td>5</td><td>6</td><td>6</td><td>7</td><td>9</td></tr><tr><td>5</td><td>6</td><td>7</td><td>7</td><td>9</td><td>10</td></tr></table>
Ignore any extras</td></tr>
<tr><td>6</td><td>(ii)</td><td></td><td>e.g.
Activities may not start on time because of delays
May need to wait for specialist equipment or other people.

[2]</td><td>B1
B1</td><td>3.5b
3.5b</td><td>May give specific examples
Delays or activities overrunning
May want to avoid splitting an activity overnight when other people are involved</td><td>Any issue relating to activity start / finish times / durations
Any resourcing issue (including availability of Sheona and Tim)</td></tr>
<tr><td>6</td><td>(iii)</td><td></td><td>Activity E cannot start until Sheona has completed A, B and D. These take 2+1+0.5 = 3.5 days, so E, cannot start until 3.5 days have elapsed.
But E is a critical activity, (so delaying E will delay the entire project).

[2]</td><td>E1

E1</td><td>2.4

1.1</td><td>

A delay in a critical activity will delay the whole project.</td><td>E.g. identifying that B and D cannot be done at the same time.

This affects critical activity E (or G)</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th colspan="2">Guidance</th></tr>
<tr><td>6</td><td>(iv)</td><td>(a)</td><td>Each is occupied for a total of 8 days,

So the longest rest either can have is 6 days</td><td>M1

A1

[2]</td><td>1.1

1.1</td><td>14 – 6 or (either) busy for 8

6
Allow if the rest for either S or T is identified as 6 days</td><td>May be implied from Answer 6

(S does ABDEGI rests for 6 days and then does K,
T does ACF rests for 6 days and then does HJK)</td></tr>
<tr><td>6</td><td>(iv)</td><td>(b)</td><td>Activity G must start at the earliest on the afternoon of on day 6 and at the latest on the morning of day 10.

Sheona is busy for 6.5 days before the end of activity G (and 1.5 days after activity G) and Tim is busy for 4 days after the end of activity G (and 4 days before the start of activity G).

If activities A to I are done as early as possible and activities J, K as late as possible then Sheona and Tim are both on a break from the afternoon of day 8 to the end of day 11

= 3.5 days</td><td>M1

A1
2</td><td>1.1

1.1</td><td>Using the timing of activity G

Or using minimum completion time = 10.5 days

Or equivalent reasoning about early/late times

3.5</td><td>May be implied from answer 3.5</td></tr>
<tr><td>6</td><td>(iv)</td><td>(c)</td><td>A, E, G, H, J, K = 3.5 days

<table><tr><td>B</td><td>C</td><td>D</td><td>F</td><td>I</td></tr><tr><td>4</td><td>5</td><td>4.5</td><td>5</td><td>4</td></tr></table></td><td>M1

A1
[2]</td><td>1.1

1.1</td><td>Activities with float = 3.5 days
A, E, G, H, J, K

Others correct</td><td>Accept with 1 or 2 errors in total</td></tr>
</table>

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